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Three-Dimensional Atom Localization and Microscopy in a Three-Level Λ-Type Atomic Configuration

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ABSTRACT

This paper introduces a method for achieving precise three-dimensional atomic microscopy within a three-level Λ-type configuration, utilizing a probe field and a standing-wave control field. The microscopy information is derived from the imaginary part of the effective susceptibility, which reveals the absorption spectrum. Applications of this approach to atomic microscopy are demonstrated for scenarios involving two atoms and interactions near boundary walls. The atomic positions are precisely controlled through parameters such as detunings (Δ), field strengths ($\beta_{1,\ p}$), decay rates ($\gamma_{a,\ b}$), and field phases (ϕ_{1y}, ϕ_{1z}). The theoretical findings presented in this study highlight the potential for significant advancements in applications such as laser cooling and nanolithography.

Keywords: Atomic Localization; Density Matrix; effective susceptibility; Atomic Microscopy

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1 Introduction

Atom microscopy is the precise measurement of an atom's position. Atom microscopy has many applications, such as neutral atom trapping and laser cooling [1,2], coherent patterning of matter waves [3], measurement of the wave function center of mass of moving atoms [4, 5], and bose-einstein condensation [6, 7]. Because of the many potential applications of atom microscopy, different methods were employed to find the position of an atom. Zubairy and colleagues [8] suggested weak probe absorption for one-dimensional (1d) atomic systems. Many approaches for 1d atom microscopy in the sub-wavelength range have been presented using excited-level population measurement [9 –12]. Qamar et al [13] proposed a method for atom microscopy based on standing wave field resonance. Wang et al [14] presented probe absorption-based 1d atomic microscopy in a four-level n-type medium. Herkommer and co-workers [15] suggested a method for one-dimensional atom microscopy considering measurement of the spontaneous spectrum known as autler-townes. Two-dimensional atomic microscopy has been widely explored, and numerous microscopy methods for two-dimensional atomic microscopy have been presented in recent years. Ivanov et al [16] established two dimensional microscopy by monitoring any upper or lower low energy state population using a four-level configuration. Wan and co-workers [17] exploited the concept of quantum interference to locate atoms in an inverted y system driven coherently. Wang and colleagues [18] proposed a (2D) atom microscopy approach founded on the double-dark resonances in an n-type system's interference. Ding et al [19] present a (2D) scheme based on an n-tripod-type five-level atomic system and an n-tripod-type 5-level medium with a coherence-controlled absorption spectrum. Ding and group members [20] also suggested a system for controlled spontaneous emission-based (2D) atomic localization in the sub-wavelength region. Qamar et al [21] devised a five-level 3d atom microscopy method using probe absorption. The weak field probe absorption spectra were used in Zhang and his colleagues' proposed method for 3D atom microscopy [22]. Ivanov et al [23] proposed (3D) microscopy by the measurement of the population of the four-level tripod atomic system. Wang and yu [24] proposed measuring probe field absorption to localize atoms in 3D space in a 3-level atomic arrangement. Zhu et al [25] suggested an ancient approach for using spatial interference in three-dimensional (3D) atom microscopy with great accuracy in a general two-level atomic system. In references [26-28], more atomic microscopy strategies in various atomic systems are provided. Atomic microscopy is an active area of research. There has been much work on the control and modification of microscopy. Many studies employing various methods and procedures have been written on the microscopy of atoms in 1D, 2D, and three-dimensions. Experimental and theoretical works on controlled and modified atom microscopy are available, some of which are mentioned above for various applications. In this paper, we study three-dimensional atom microscopy by probe absorption spectra in a three-level λ-type system. We investigate spherical and wall-like microscopy in different positions by tuning the detunings Δ_{p1}, direction η_1, and phases $\varphi_{1y,\ 1z}$ parameters of the applied field. This work offers significant advantages over previous studies, as it explores spherical, cylindrical, and wall-like localization in a well-controlled manner. The findings presented in this article demonstrate promising applications in nanolithography, laser cooling, and Bose-Einstein condensation.

2 Model and dynamics

This section presents the model and dynamics of a three-level Λ-type atomic configuration, as illustrated in Figure 1. This system is chosen for its simplicity and ease of experimental implementation. The configuration comprises of two ground states |a⟩ and |b⟩ and an excited state |c⟩. A probe field having Rabi frequency of β_p and a detuning of Δ_p connects the state |a⟩ to |c⟩. Similarly, the state |b⟩ is coupled to |c⟩ by a controlled field with a Rabi frequency of β_1 and a detuning of Δ_1. The decay rate between states |c⟩ and |a⟩ is denoted as γ_{ca}, and the decay rate between states |c⟩ and |b⟩ is denoted as γ_{cb}.

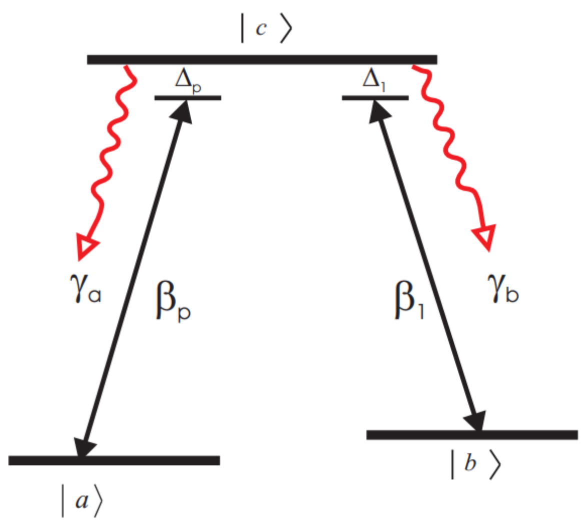

Figure 1: A three-level atomic medium

The interaction Hamiltonian given as;

$$H_I = -\frac{\hbar}{2}[\beta_p e^{-i\Delta_p t}|a\rangle\langle c| + \beta_c e^{-i\Delta_1 t}|b\rangle\langle c|] + H.C \tag{1}$$

In this case, H.C. denotes the Hamiltonian conjugate of the system. Additionally, $\Delta p = \omega_{ac} - \omega_p$ and $\Delta_1 = \omega_{bc} - \omega_1$. To examine the system's dynamics, we employ the density matrix formalism.

$$\dot{\hat{\rho}} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}] - \frac{1}{2}\sum\gamma_{ij}(\mathcal{L}\mathcal{L}^\dagger\rho + \rho\mathcal{L}\mathcal{L}^\dagger - 2\mathcal{L}^\dagger\rho\mathcal{L}) \tag{2}$$

In this equation, the first term represents the interaction part, while the second term corresponds to the decay process. The operators L † and L denote the raising and lowering operators, respectively. This equation is utilized to study the dynamics of the system and derive the coupled rate equations governing its evolution.

$$\dot{\tilde{\rho}}_{ac} = \mathcal{F}_1\tilde{\rho}_{ac} + \frac{i}{2}\beta_p\tilde{\rho}_{cc} - \frac{i}{2}\beta_p\tilde{\rho}_{aa} - \frac{i}{2}\beta_1\tilde{\rho}_{ab} \quad (3)$$

$$\dot{\tilde{\rho}}_{ab} = \mathcal{F}_2\tilde{\rho}_{ab} + \frac{i}{2}\beta_p\tilde{\rho}_{cb} - \frac{i}{2}\beta_1^*\tilde{\rho}_{ac} \quad (4)$$

$$\mathcal{F}_1 = i\Delta_p - \frac{1}{2}\gamma_a$$

$$\mathcal{F}_2 = i(\Delta_p - \Delta_1) - \frac{1}{2}(\gamma_a + \gamma_b)$$

Initially, the system is in the ground state |a⟩, and the populations of other states |b⟩ and |c⟩ are zero. Using the approach of perturbation, we can obtain the solution of the steady-state of ρ_{ac}

$$\tilde{\rho}_{ac} = -\frac{2i\mathcal{F}_1\beta_p}{4\mathcal{F}_1\mathcal{F}_2+\beta_1^2} \quad (5)$$

The electric susceptibility is given as:

$$\chi = \frac{2N\mathcal{U}_{ac}^2}{\epsilon_0\hbar\beta_p}\tilde{\rho}_{ac} \quad (6)$$

Where;

$$\mathcal{U}_{ac} = \sqrt{\frac{3\hbar\gamma_a\epsilon c^3}{2\omega^3}} \quad (7)$$

The Rabi frequency of control given as;

$$\beta_1 = \mathcal{R}_1(\sin[\alpha_1 kx] + \sin[\alpha_1 ky + \phi_{1y}] + \sin[\alpha_1 kz + \phi_{1z}]) \quad (8)$$

3 Result and Discussion

This section presents the results and discussion of the model for 3D atom microscopy, the plots are traced by plotting the imaginary part of susceptibility (χ) for position coordinates kx, ky, and kz, which carries information about the position of atoms. The position ranges $-\pi \leq kx, ky, kz \leq \pi$. We take the decay rate $\gamma = 10^9$, number of atoms $n = 10^{12}$, permittivity of free space $\epsilon = 8.85\times10^{-12}$F/m, reduce plank's constant $\hbar = 1.02\times10^{-34}$Js, angular frequency $\omega = 1000\gamma$, $\lambda = 2\pi c\ \omega$, speed of light in vacuum $c = 3 \times 10^8$m/s.

The results indicate that atomic localization is highly sensitive to detunings, decay rates, and applied field strengths. By varying these parameters, the localization patterns can be controlled to exhibit spherical, wall-like, or cylindrical geometries.

In Figure 2, the imaginary part of electric susceptibility is plotted for positions kx, ky, and kz. Two atoms are investigated by taking the parameters: strength $\mathcal{R}_1 = 3\gamma$, phases $\phi_{1y} = 2\pi$, $\phi_{1z} = \pi$ and decay rates. $\gamma_1 = \gamma_2 = 0.1\gamma$, direction $\alpha_1 = -1$, detuning $\Delta_1 = 0.391\gamma$, $\Delta_p = 5\gamma$ One of the two atoms is observed at the bottom of the first quadrant, while the other is at the top of the third quadrant, as given in Figure 2.a. By tuning the phases to $\phi_{1y} = \phi_{1z} = 2\pi$, the position of atoms changes, and one atom shifts to the top of the first quadrant while the other shifts to the bottom of the third quadrant, as given in Figure 2.b. The position of atoms is investigated at the bottom of the second and top of the fourth quadrants by choosing $\phi_{1y} = \pi$ and $\phi_{1z} = 2\pi$, as given in Figure 2.c.

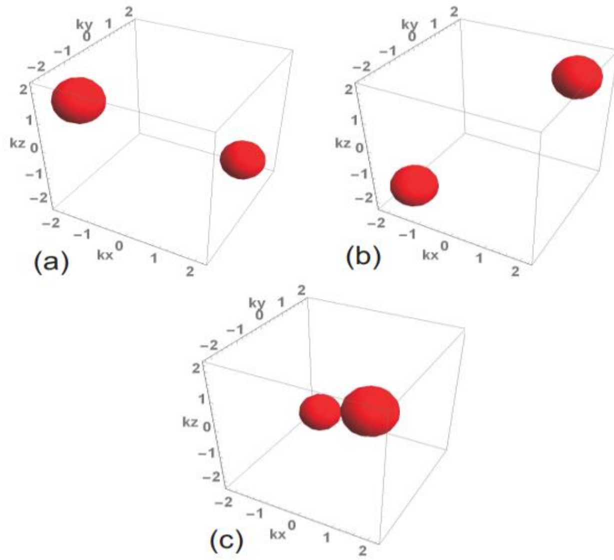

Figure 2: Spherical microscopy by plotting imaginary part of susceptibility

In Figure 3, the plots are traced for walls-like atom microscopy by plotting the imaginary part of susceptibility by tuning the strength $\mathcal{R}_1 = 5\gamma$, phases $\phi_{1y} = \pi$, $\phi_{1z} = \frac{\pi}{2}$, decay rates $\gamma_1 = \gamma_2 = 1\gamma$, direction $\alpha_1 = 0.01$, detuning's $\Delta_1 = 9.1 \times 10^{-4}\gamma$, $\Delta_p = 5\gamma$, three walls-like atom microscopy is reported as shown in Fig 3. a. By changing the phases to $\phi_{1y} = \frac{\pi}{2}$, $\phi_{1z} = \frac{\pi}{3}$, and detuning's $\Delta_p = 10\gamma$ the direction of walls-like atom microscopy changes, as shown in Fig 3. b. By selecting $\phi_{1y} = \frac{3\pi}{2}$, $\phi_{1z} = \frac{\pi}{2}$, detuning's $\Delta_p = 15\gamma$, detuning's $\Delta1 = -1\gamma$, six walls-like localization three in the right side and three in the left side of the box as shown in Figure 3.c.

This study highlights that atomic localization is highly sensitive to the strength of the applied control fields. Increasing the field strength causes the spherical localization pattern to transform into a wall-like localization structure.

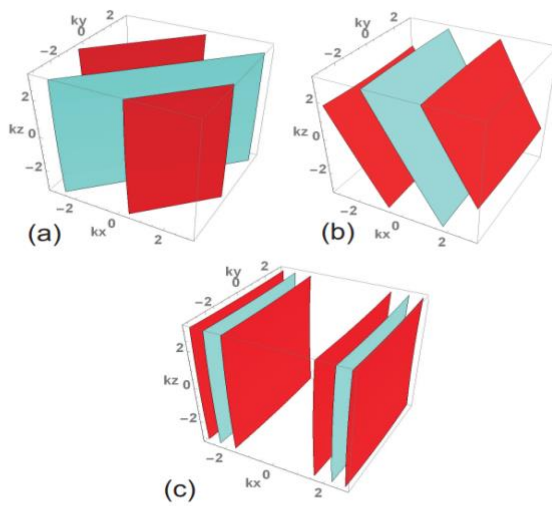

Figure 3: Walls microscopy by plotting imaginary part of susceptibility

In Figure 4, the imaginary part of electric susceptibility is plotted for positions kx, ky, and kz. Wall-like atom microscopy investigated by taking the parameters, strength $\mathcal{R}_1 = 5\gamma$, phases $\phi_{1y} = \phi_{1z} = \frac{3\pi}{2}$, decay rates $\gamma_1 = \gamma_2 = 1\gamma$, direction $\alpha_1 = 0.01$, detuning's $\Delta_1 = -1\gamma$, $\Delta_p = 15\gamma$ three wall like atoms microscopy are observed as given in Figure 4.a. Similarly four Walls-like localization investigated by taking the parameters, strength $R_1 = 5\gamma$, phases $\phi_{1y} = \phi_{1z} = \frac{\pi}{2}$, decay rates $\gamma_1 = \gamma_2 = 1\gamma$, direction $\alpha_1 = 0.01$, detuning's $\Delta_1 = 9.1\times10^{-4}\gamma$, $\Delta_p = 5\gamma$ as shown in the Fig 4.b. Three wall like atom microscopy is reported across the diagonal as given in Fig 4.c. by taking strength $\mathcal{R}_1 = 3\gamma$, phases $\phi_{1y} = \frac{\pi}{3}$, $\phi_{1z} = \frac{\pi}{2}$, decay rates $\gamma_1 = \gamma_2 = 1\gamma$, direction $\alpha_1 = 0.01$, detunings $\Delta_1 = 3.51 \times 10^{-1}\gamma$, $\Delta_p = 5\gamma$.

In summary, walls provide structural means for confinement, while spherical localization ensures uniform and isotropic trapping conditions, creating an ideal platform for exploring fundamental and applied aspects of Bose-Einstein condensates.

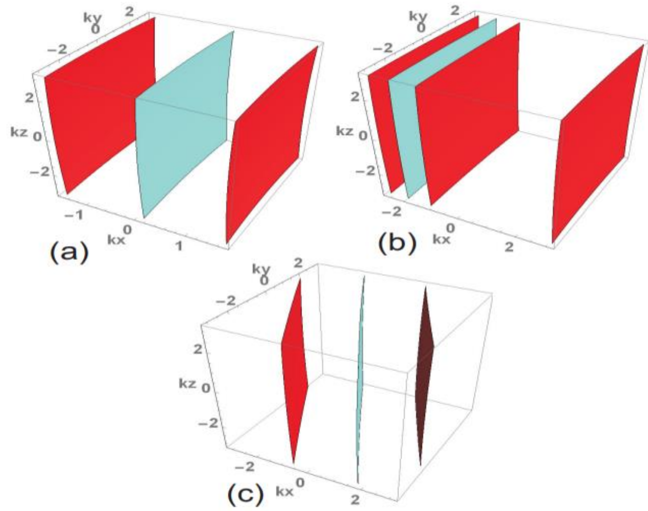

Figure 4: Wall-like atomic microscopy by plotting imaginary part of susceptibility

4 Conclusion

In conclusion, we investigated three-dimensional atom microscopy by absorption spectrum using density matrix formalism in a three-level Λ-type atomic medium with precise position and probability measurement within a wavelength domain along the x, y, and z planes. Two atom microscopy is reported by different parameters of the applied control field. The position of atoms is controlled by direction and phases of the field. In addition to spherical atom microscopy, walls, like atom microscopy, are investigated by different parameters, directions, and phases of the control field. The study's precise control of atomic positions enables high resolution patterning in nanolithography, facilitating advanced nano-device fabrication. In laser cooling, the tunable parameters enhance trapping and cooling efficiency, particularly in dense atomic ensembles or Bose-Einstein condensates.

Author Contributions

Najm Uddin, Attaullah Khan: Methodology, Formal Analysis, Writing – Original Draft Preparation, Writing – Review & Editing.

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Conflict of Interests

This work does not have any potential conflicts of interest.

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Declaration Statement of Generative AI

The author declare they have not used Artificial Intelligence (AI) tools in the creation of this article

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